Population Mathematics

Evolution works and life is necessarily a struggle, fundementally, because of the mathematics of population growth. The mathematics are unfortunately somewhat complicated. The ramifications of the mathematics are terribly important when looking at almost all questions of human existence. But, since the mathematics are not universally understood, many people put forward ideologies and politics that ignore the harsh realities of the following equations.

One of my physics book from my first year physics class (Physics, for Scientists & Engineers, Third Edition, Raymond Serway) contained a number of essays written by contributing authors about various topics related to physics or mathematics. One of these essays was entitled "Exponential Growth" by Albert Bartlett of the University of Colorado. In the essay Prof. Bartlett describes exponential growth using a number of examples such as compound interest of savings accounts and annual increases in world population.

He states that based on ìreportsî the world population in July 1987 hit 5 billion; that there were 28 births per 1000 people per year and that there were 11 deaths per 1000 people per year. He then derived an equation which estimated the world population at any point in time assuming that the growth rate of 1.7% (28 ñ 11 = 17 per 1000 = 1.7%) remained constant.

$$P(t) = (5,000,000,000)e^{ln(1.017)t}$$

Where t is the number of years since 1987.

If you have a basic knowledge of Calculus and/or are interested in the derivation of that equation read the following, otherwise skip:

> Certain values change at a rate proportional to their current value. For example, money in a bank account increases by an amount that is proportional to how much money is currently in the account. Populations also increase at a rate that is proportional to the number of individuals in a population. In mathematics this is denoted as:
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> $$\frac{d}{dt}P(t) = \alpha P(t)$$
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> which is an ordinary differential equation, which states that the rate of change of a population is proportional to the size of that population.
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> In algebra variables represent numbers. When you look for a solution for the variable x, you are looking for the number that solves the equation (i.e., makes the equation true). For differential equations, the variables rather than representing a number represent a function. (In this case the function we are looking for is P(t)).
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> In actuality the notation above while being the standard mathematics notation is fairly weak. For example d/dt represents an operation, P(t) represents a variable function (that varies with time, t) and ëαí represents a constant. But, a mathematician would only know all of that because they are familiar with the conventions of mathematics, not because a strict set of notational rules are being followed.
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> Generally, there is not some recipe or mechanical operation that one can follow in order to solve differential equations. Often times, one must simply guess or use trial and error in order to discover a solution to a problem.
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> d/dt represents the derivative of a function. The derivative of a function is a function which gives the slope of the original function at any given point. Perhaps the easiest way to think about derivatives is to relate it to driving. While driving a car, the odometer shows the distance that a car has traveled. The speedometer shows the velocity with which the car is moving. This velocity is the rate at which the odometer is increasing. The odometer is some function of time which gives how far the car has moved at that point in time. The speedometer is the derivative of the odometer, giving the rate at which the odometer is increasing at any point in time.
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> There exists a function whose derivative is equal to the function itself (i.e., $df(x)/dx = f(x)$) , which is: $f(x) = e^x$. Where e is a mathematical constant known as Euler's number and is approximately equal to 2.17828. Our differential equation is very similar to the above differential equation, so through a little trial and error we can see that:
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> $$P(t) = P_0 e^{\alpha t}$$

Because $\frac{d}{dt}P(t) = \alpha P_0 e^{\alpha t} = \alpha P(t)$, which is exactly what we were looking for. (Understanding this, requires knowledge of how to calculate derivatives, which is covered in Calculus.)

The next thing we need to do is solve for the constants α and P_0. If we define the time 0 to correspond to 1987, since we know that the world population was 5 billion then we can figure out P_0:

$$P(0) = 5,000,000,000 = P_0 e^0 = P_0$$

In order to calculate α we can use the 1.7% annual growth rate in order figure out what P(1) is.

$$P(1) = 1.017P_0 = P_0 e^{\alpha}$$
$$e^{\alpha} = 1.017$$
$$\alpha = ln(1.017)$$

Which gives us:

$$P(t) = (5,000,000,000)e^{ln(1.017)t}$$

Prof. Bartlett then uses this equation to calculate some astounding things:

When did Adam and Eve live?

$$2 = (5,000,000,000)e^{ln(1.017)t}$$
$$t = \frac{ln(2/(5,000,000,000))}{ln(1.017)} = -1283$$ = 703 AD

When will world population equal one person per square meter? Land area of continents (excluding Antarctica) 1.24×10^{14} square meters

$$1.24 \times 10^{14} = (5,000,000,000)e^{ln(1.017)t}$$
$$t = \frac{ln(1.24 \times 10^{14}/(5,000,000,000))}{ln(1.017)} = 600$$ years from 1987

When will the mass of human population equal the mass of the earth? Assume the mass of the average person to be 65 kg, the mass of earth is 5.98×10^{24} kg

$$(5.98 \times 10^{24})/65 = (5,000,000,000)e^{ln(1.017)t}$$
$$t = \frac{ln(5.98 \times 10^{24}/65/(5,000,000,000))}{ln(1.017)} = 1,811$$ years from 1987

Iíve always thought that those calculations were pretty amazing and quite strong arguments that population growth is a problem.

However, who knows what the world is going to be like 1,811 years from now? We may have populated numerous other planets or solar systems. So how do we know for sure that having a human mass equal to the mass of the earth is going to be a problem?

I was trying to come up with another calculation that would be even stronger than the examples provided by Prof. Bartlett. Here is what I came up with:

Imagine that each human being is given one cubic meter of space to live in. Everything they need to survive, food, shelter etc is contained in that single cubic meter. We could take the entire human population and simply stack them up into a massive sphere of humanity.

The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

The radius is therefore:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

in 1987 the volume of that sphere would be 5,000,000,000 cubic meters, which would be a radius of:

$$r = \sqrt[3]{\frac{3(5,000,000,000)}{4\pi}} = 1,061$$ meters

the radius as a function of time would be:

$$r(t) = \sqrt[3]{\frac{3P(t)}{4\pi}}$$

the velocity of the radius would simply be the derivative:

$$v = \frac{d}{dt}r(t) = \frac{1}{3}\left[\frac{3P(t)}{4\pi}\right]^{-2/3}\frac{\alpha 3P(t)}{4\pi}$$

therefore time as a function of velocity would be:

$$v = \left[\frac{4\pi}{3P(t)}\right]^{2/3}\frac{\alpha P(t)}{4\pi}$$
$$v = \alpha\sqrt[3]{\frac{P(t)}{9\times 4\pi}}$$
$$v = \alpha\sqrt[3]{\frac{P_0 e^{\alpha t}}{9\times 4\pi}}$$
$$t(v) = \frac{1}{\alpha}ln\left(\frac{36\pi v^3}{\alpha^3 P_0}\right)$$

the speed of light measured in meters per year is:

$$c = 2.998\times 10^8(3600)(24)(365.24)$$

The question is how long before the human population sphere is increasing at a rate greater than the speed of light:

$t(c) = 6,229$ years

In order for humans to continue to increase at 1.7% annually within 6229 years we will have to break a fundamental law of physics, namely that it is not possible to travel at the speed of light. For comparison sake, the age of the universe is about 12 billion years, age of the Earth is about 4.5 billion years, age of life on Earth about 4 billion years, birth of dinosaurs about 237 million years ago, death of dinosaurs about 65 million years ago, birth of human about 300,000 years ago, start of civilization about 12,000 years ago. In other words, 6229 years is nothing.

The clear conclusion of this result is that the human population will not be increasing continually at 1.7% for the next 6229 years. There are only 2 ways for the population expansion rate to be reduced: decrease the number of births or increase the number of deaths.

Recently, it has been noted that the population expansion rate in many western countries and in China has decreased to the point where the populations of these nations is decreasing over time. This fact has been put forth as evidence that there is no world population problem.Let's say this assessment is correct and about half the worldís population is living in nations thatís population will eventually go to 0. That still leaves half the world living in nations where the population is increasing. Letís simply call the population of the Earth in 1987, 2.5 billion instead of 5 billion and then letís recalculate our speed of light point.

$t(c) = 6,270$ years

By effectively eliminating half of the population, the speed of light scenario is delayed by 41 years. Which to me says: there still is a population problem.